

# Game-theoretic Risk Assessment for Distributed Systems (GRADS)

**Sponsor: DASD(SE)**

**By**

**Dr. Paul T. Grogan, Ambrosio Valencia-Romero, Abbas Ehsanfar**

**9<sup>th</sup> Annual SERC Sponsor Research Review**

**November 8, 2017**

**FHI 360 CONFERENCE CENTER**

**1825 Connecticut Avenue NW**

**8th Floor**

**Washington, DC 20009**

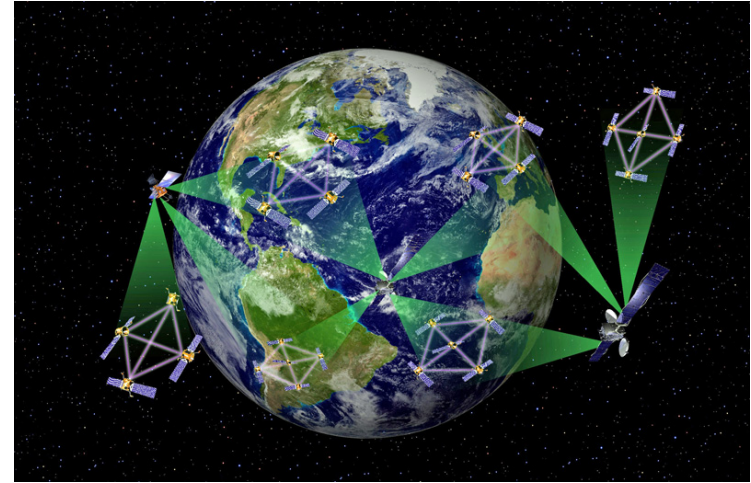
**[www.sercuarc.org](http://www.sercuarc.org)**

## Upside Potential

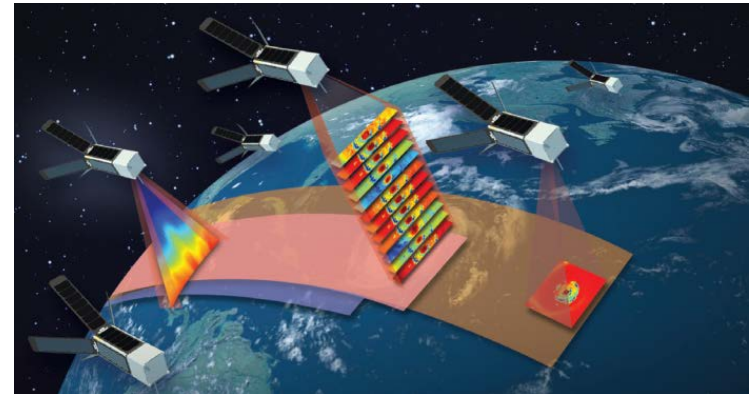
- Flexibility, robustness
- Resource efficiency – decentralization theorem

## Downside Risk

- Interdependencies leading to cascading failures
- “Robust-yet-fragile” (Alderson and Doyle, 2010)



System F6 Concept (DARPA)



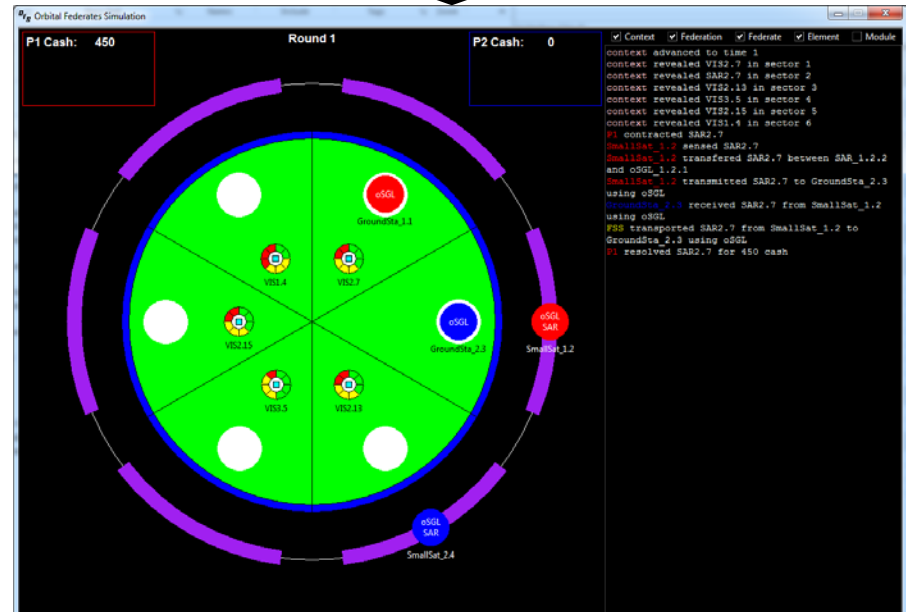
TROPICS Mission Concept (NASA, Lincoln Labs)

- **How to assess the strategic risk of distributed systems?**
  - Tradeoff between expected upside and possible downside
  - Strategic risk: decision stability (*NOT* uncertainty in value)
  - Evaluate an objective risk metric based on Selten's (1995) Weighted Average Log Measure (WALM) of risk dominance
- Use a stylized model of federated satellite systems:
  - Build on existing work with  $n = 2$  symmetric players:
    - P.T. Grogan, K. Ho, A. Golkar, and O.L. de Weck, "Multi-actor Value Modeling for Federated Systems," *IEEE Systems*, 2016. Early access.
  - Investigate asymmetric and  $n > 2$  player strategic design games
  - Develop method in simplified *Orbital Federates* design context

Board game to study collective decisions in federated satellite systems (Grogan and de Weck, 2015)

Python simulation (OFS-PY)  
[github.com/ptgrogan/ofspy](https://github.com/ptgrogan/ofspy)

Design and strategy decisions, RNG seed



Initial cost and final value outcomes

**Monolithic ( $\phi$ ) Strategy:** no interaction between players

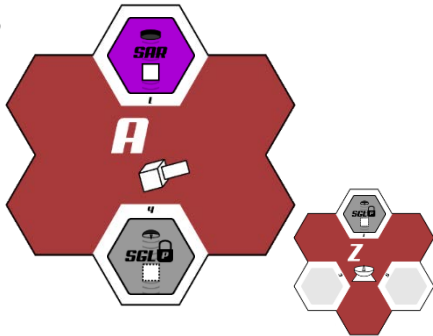
$d_1^\phi$



$$v_1^\phi = 0$$

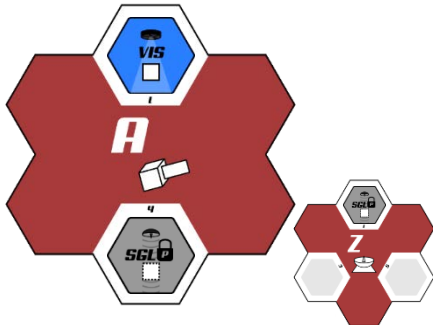
*NPV with 1000 samples, 2% discount rate*

$d_2^\phi$



$$v_2^\phi = -39$$

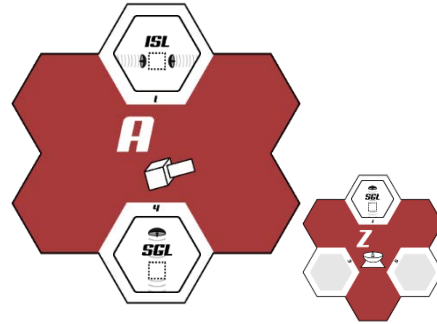
$d_3^\phi$



$$v_3^\phi = 30$$

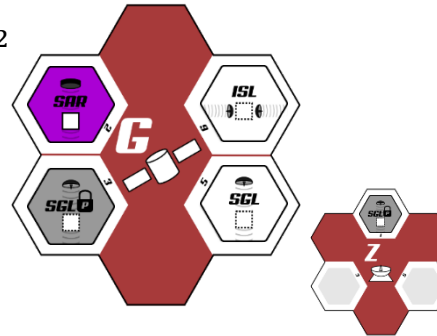
**Distributed ( $\psi$ ) Strategy:** opportunistic data exchange: 100 for space-to-ground link (SGL), 100 for inter-satellite link (ISL)

$d_1$



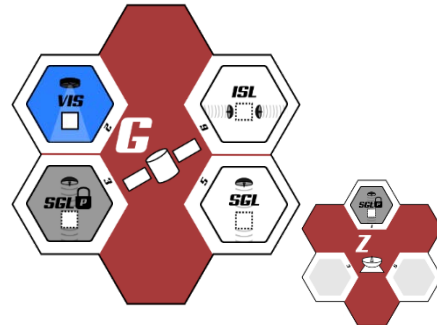
$$V_1^\psi(d_1, d_2, d_3) = 100$$

$d_2$



$$V_2^\psi(d_1, d_2, d_3) = 536$$

$d_3$



$$V_3^\psi(d_1, d_2, d_3) = 639$$



# Foundation: Stag Hunt Game

	Hare ( $\phi$ )	Stag ( $\psi$ )
Hare ( $\phi$ )	2 2	4 0
Stag ( $\psi$ )	0 4	5 5

- Symmetric coordination game
- Two pure Nash equilibria:
  - Risk-minimizing (Hare)
  - Payoff-maximizing (Stag)
- How to select strategy?



*Cy deuse comment on doit aler laister pour le corf* Bibliothèque nationale de France

Stag hunt by Gaston Phoebus  
(Bibliothèque Nationale de France)

- Harsanyi and Selten (1972): *Generalized Nash Solution for Two-person Bargaining Games*
  - Efficient solutions maximize *product* of cooperative gains
  - Generalized solution extends to incomplete information
- Harsanyi and Selten (1988): *A General Theory of Equilibrium Selection in Games*
  - “Tracing” procedure to select risk dominant equilibrium
- Selten (1995): *Axiomatic Theory of Risk Dominance for Bipolar Games with Linear Incentives*
  - Scalar measure to assess risk dominance

- Risk dominance based on concept of “deviation loss”
- Deviation loss measures utility loss if a player moves away from an equilibrium

$$L_i(\phi) \propto 2 - 0 = 2$$

$$L_i(\psi) \propto 5 - 4 = 1$$

- Deviation loss ratio measures relative losses between two equilibria

$$\lambda_i(\phi, \psi) = \frac{L_i(\phi)}{L_i(\psi)} = \frac{2}{1} = 2$$

	Hare ( $\phi$ )	Stag ( $\psi$ )
Hare ( $\phi$ )	2 2	4 0
Stag ( $\psi$ )	0 4	5 5



- Normalized deviation losses

$$u_i = \frac{L_i(\phi)}{L_i(\phi) + L_i(\psi)}$$

- Influence weights  $w_i(A)$  measure the importance of one player on others' stability

- Weighted average log measure (WALM) of risk:

$$R_\psi = \sum_{i=1}^n w_i(A) \ln \left( \frac{u_i}{1 - u_i} \right)$$

—  $R_\psi > 0$ :  $\phi$  is “risk dominant”

—  $R_\psi < 0$ :  $\psi$  is “risk dominant”

	Hare ( $\phi$ )	Stag ( $\psi$ )
Hare ( $\phi$ )	2 2	4 0
Stag ( $\psi$ )	0 4	5 5

$$\begin{aligned} R(u, A) &= 0.5 \ln \left( \frac{2}{1} \right) + 0.5 \ln \left( \frac{2}{1} \right) \\ &= \ln \left( \frac{2 - 0}{5 - 4} \right) = 0.69 \end{aligned}$$

- Hare is risk dominant equilibrium in this game

- Can maximize value under independent strategy

$$\mathcal{V}_i^\phi = \max_{d \in \mathcal{D}_i} V_i(d_i)$$

- Collective value is a function of candidate designs  $d_i, \dots$

$$V_i(d_i), \quad V_i^\psi(d_i, \dots)$$

	Indep. ( $\phi$ )	Collect. ( $\psi$ )
Indep. ( $\phi$ )	$\mathcal{V}_1^\phi$	$\mathcal{V}_1^\phi$ $\mathcal{V}_2^\phi$ $V_2(d_2)$
Collect. ( $\psi$ )	$V_1(d_1)$	$V_1^\psi(d_1, d_2)$ $\mathcal{V}_2^\phi$ $V_2^\psi(d_1, d_2)$

$$R_\psi(d_1, d_2) = \sum_{i=1}^2 \frac{1}{2} \ln \left( \frac{\mathcal{V}_i^\phi - V_i(d_i)}{V_i^\psi(d_1, d_2) - \mathcal{V}_i^\phi} \right)$$

# $n = 3$ Asymmetric Design Games

	Independ. ( $\phi$ )	Collect. ( $\psi$ )		Independ. ( $\phi$ )	Collect. ( $\psi$ )
Independ. ( $\phi$ )	$v_1^\phi$ $v_2^\phi$ $v_3^\phi$	$v_1^\phi$ $V_2(d_2)$ $v_3^\phi$	Independ. ( $\phi$ )	$v_1^\phi$ $v_2^\phi$ $V_3(d_3)$	$v_1^\phi$ $V_2^\psi(d_2, d_3)$ $V_3^\psi(d_2, d_3)$
Collect. ( $\psi$ )	$V_1(d_1)$ $v_2^\phi$ $v_3^\phi$	$V_1^\psi(d_1, d_2)$ $V_2^\psi(d_1, d_2)$ $v_3^\phi$	Collect. ( $\psi$ )	$V_1^\psi(d_1, d_3)$ $v_2^\phi$ $V_3^\psi(d_1, d_3)$	$V_1^\psi(d_1, d_2, d_3)$ $V_2^\psi(d_1, d_2, d_3)$ $V_3^\psi(d_1, d_2, d_3)$

Independ. ( $\phi$ )

Collect. ( $\psi$ )

$$R_\psi(d_1, d_2, d_3) = \sum_{i=1}^3 w_i(A) \ln \left( \frac{v_i^\phi - V_i(d_i)}{V_i^\psi(d_1, d_2, d_3) - v_i^\phi} \right)$$

**Monolithic ( $\phi$ ) Strategy:** no interaction between players

$d_1^\phi$

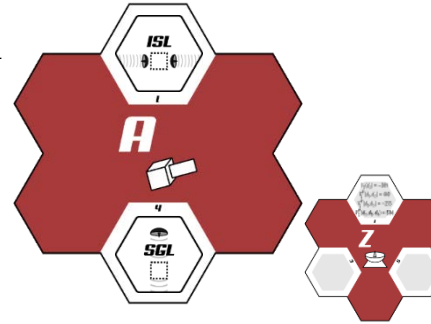


$$v_1^\phi = 0$$

NPV with 1000 samples, 2% discount rate

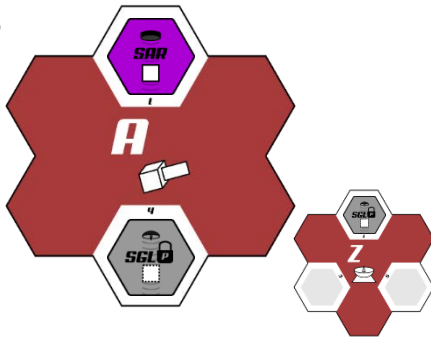
**Distributed ( $\psi$ ) Strategy:** opportunistic data exchange: 100 for space-to-ground link (SGL), 100 for inter-satellite link (ISL)

$d_1$



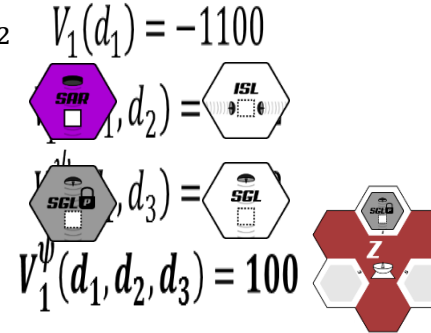
$$\begin{aligned} V_1(d_1) &= -1100 \\ V_1^\psi(d_1, d_2) &= -572 \\ V_1^\psi(d_1, d_3) &= -653 \\ V_1^\psi(d_1, d_2, d_3) &= 100 \end{aligned}$$

$d_2^\phi$



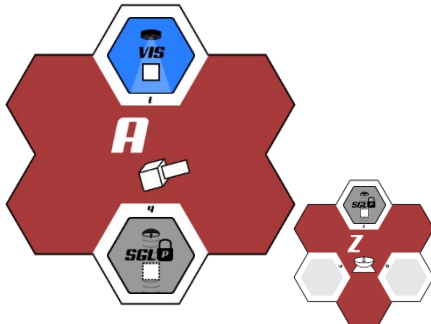
$$v_2^\phi = -39$$

$d_2$



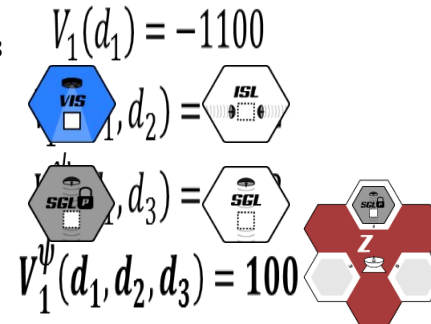
$$\begin{aligned} V_2(d_2) &= -389 \\ V_2^\psi(d_1, d_2) &= 440 \\ V_2^\psi(d_2, d_3) &= -255 \\ V_2^\psi(d_1, d_2, d_3) &= 536 \end{aligned}$$

$d_3^\phi$



$$v_3^\phi = 30$$

$d_3$



$$\begin{aligned} V_3(d_3) &= -320 \\ V_3^\psi(d_1, d_3) &= 386 \\ V_3^\psi(d_2, d_3) &= -158 \\ V_3^\psi(d_1, d_2, d_3) &= 639 \end{aligned}$$

# Strategic Design Game A

	Mono. ( $\phi$ )	Distrib. ( $\psi$ )		Mono. ( $\phi$ )	Distrib. ( $\psi$ )
Mono. ( $\phi$ )	0 -39 30	0 -389 30	Mono. ( $\phi$ )	0 -39 -320	0 -255 -158
Distrib. ( $\psi$ )	-1100 -39 30	-572 440 30	Distrib. ( $\psi$ )	-652 -39 386	100 536 639
	Mono. ( $\phi$ )			Distrib. ( $\psi$ )	

$$\ln\left(\frac{u}{1-u}\right) = \begin{bmatrix} 2.40 \\ -0.50 \\ -0.55 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0.533 & 0.467 \\ 0.876 & 0 & 0.124 \\ 0.783 & 0.217 & 0 \end{bmatrix}, \quad w(A) = \begin{bmatrix} 0.455 \\ 0.296 \\ 0.249 \end{bmatrix}$$

$$R_\psi = 0.455 \cdot 2.40 - 0.296 \cdot 0.50 - 0.249 \cdot 0.55 = \mathbf{0.80}$$

- Players do not have equal influences on stability
  - Player 1 is most influential on stability, sees  $\psi$  as very unstable
  - Players 2 and 3 have similar influence on stability, see  $\psi$  as more stable
- $R_\psi > 0$ : Monolithic/independent strategy  $\phi$  is “risk dominant”



**Monolithic ( $\phi$ ) Strategy:** no interaction between players

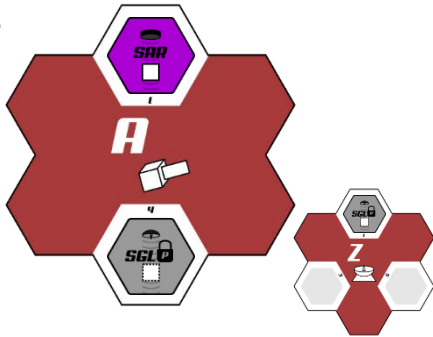
$d_1^\phi$



$$v_1^\phi = 0$$

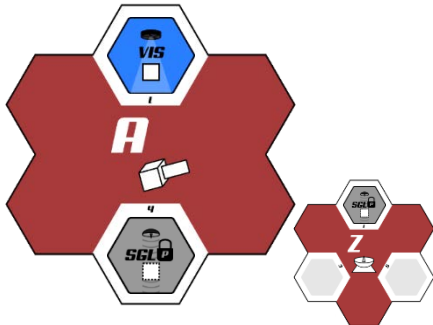
NPV with 1000 samples, 2% discount rate

$d_2^\phi$



$$v_2^\phi = -39$$

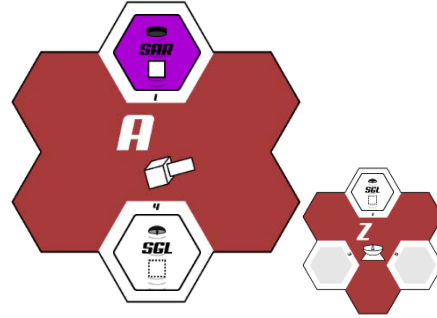
$d_3^\phi$



$$v_3^\phi = 30$$

**Distributed ( $\psi$ ) Strategy:** opportunistic data exchange: 100 for space-to-ground link (SGL), 100 for inter-satellite link (ISL)

$d_1$



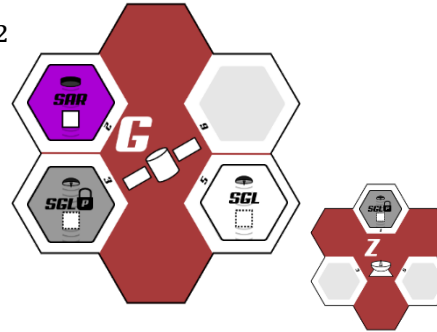
$$V_1(d_1) = -104$$

$$V_1^\psi(d_1, d_2) = 83$$

$$V_1^\psi(d_1, d_3) = 33$$

$$V_1^\psi(d_1, d_2, d_3) = 271$$

$d_2$



$$V_2(d_2) = -288$$

$$V_2^\psi(d_1, d_2) = 141$$

$$V_2^\psi(d_2, d_3) = -288$$

$$V_2^\psi(d_1, d_2, d_3) = 150$$

$d_3$



$$V_3(d_3) = -220$$

$$V_3^\psi(d_1, d_3) = 391$$

$$V_3^\psi(d_2, d_3) = -161$$

$$V_3^\psi(d_1, d_2, d_3) = 378$$

# Strategic Design Game B

	Mono. ( $\phi$ )		Distrib. ( $\psi$ )			Mono. ( $\phi$ )		Distrib. ( $\psi$ )	
Mono. ( $\phi$ )	0	-39	0	-288	Mono. ( $\phi$ )	0	-39	0	-288
Distrib. ( $\psi$ )	-104	-39	83	141	Distrib. ( $\psi$ )	33	-39	271	150
	30	30	30	30		-220	-161	391	378
	Mono. ( $\phi$ )					Distrib. ( $\psi$ )			

$$\ln\left(\frac{u}{1-u}\right) = \begin{bmatrix} -0.96 \\ 0.28 \\ -0.33 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0.566 & 0.434 \\ 0.989 & 0 & 0.011 \\ 0.961 & 0.039 & 0 \end{bmatrix}, \quad w(A) = \begin{bmatrix} 0.494 \\ 0.288 \\ 0.218 \end{bmatrix}$$

$$R_\psi = -0.494 \cdot 0.96 + 0.288 \cdot 0.28 - 0.218 \cdot 0.33 = -0.46$$

- Players do not have equal influences on stability
  - Player 1 is most influential on stability, sees  $\psi$  as more stable
  - Players 2 and 3 have similar influence on stability
  - Player 2 sees  $\phi$  as more stable, player 3 sees  $\psi$  as more stable
- $R_\psi > 0$ : Distributed/collective strategy  $\psi$  is “risk dominant”

## Design Scenario A

- High(er) payoffs for distributed system concepts
  - Player 1 (most influential) is at unstable local equilibrium: fears large losses of uncorroborated collective action
  - Players 2 and 3 are more stable, but not significant enough to change strategic risk
- Strategic risk assessment: monolithic system is more stable

## Design Scenario B

- Low(er) payoffs for distributed system concepts
  - Player 1 (most influential) is at stable local equilibrium: benefits from independent value generation
  - Player 2 is at unstable local equilibrium, but not significant enough to change strategic risk
- Strategic risk assessment: distributed system is more stable

## Applications to Systems Engineering practice:

- Tradespace analysis
  - Assess WALM risk dominance for enumerated design space
  - Eliminate high-payoff but unstable design alternatives
- Strategy analysis
  - Assess WALM risk dominance for collective strategies (operational rules for player interactions)
  - Identify “good” strategies which stabilize high-payoff designs

## Necessary steps to advance GRADS research:

- Theoretical extensions:
  - Currently violates “linear incentives” assumption
  - How to practically but rigorously quantify deviation losses?
- Practical extensions:
  - Develop multi-actor value function for real-world system (space or non-space domain)
  - Assess WALM risk of alternative architectures if a bipolar game

Paul T. Grogan

[pgrogan@stevens.edu](mailto:pgrogan@stevens.edu)